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Math 12 Honours: Section 5.2 What are Logs and Basic with Logarithm

$\log(A \times B) = \log A + \log B$ $\qquad \log\left(\frac{A}{B}\right) = \log A - \log B$ $\qquad \log A^n = n \log A$ $\qquad \log_a b^c = \frac{\log b^c}{\log a}$

1. Rewrite each of the following in exponential form:

a) $\log_3 81 = 4$ $3^4 = 81$	b) $\log_4 2048 = b$ $4^b = 2048$	c) $-\log_5 a = 13$ $5^{-13} = a$
d) $\log_c d = e$ $c^e = d$	e) $\log_{2x} 100 = 5$ $2x^5 = 1000$	f) $\log_{\sqrt{3}} a = b$ $\sqrt{3}^{\,b} = a$

2. Evaluate each of the following without using a calculator:

a) $\log_2 8^3$ $= \log_2 2^9 = \boxed{9}$	b) $\log_3 \sqrt{243}$ $\log_3 3^{\frac{5}{2}} = \boxed{\frac{5}{2}}$	c) $\log 100 + \log_3 \sqrt{3}$ $= 2 + \frac{1}{2} = \boxed{\frac{5}{2}}$
d) $3\log_{0.125} 32$ $= \log_{2^{-3}} 2^{15} = \frac{15}{-3} = \boxed{-5}$	e) $\log_2 \left(\log(0.0001)\right)^2$ $= \log_2 (-4)^2 = \boxed{4}$	f) $\log_8 \left(\frac{\sqrt{256} \times 64}{\sqrt[3]{1024}}\right)$ $\log_{2^3} \left(\frac{2^{6+4}}{2^{\frac{10}{3}}}\right) = \frac{10 - \frac{10}{3}}{3} = \boxed{\frac{20}{9}}$
g) $\log_5 25 \times \log_5 0.2 \times \log_{0.2} 125$ $= 2 \times (-1) \times (-3)$ $= \boxed{6}$	h) $\log_{12} 16 + \log_{12} 9$ $\log_{12} 16 \cdot 9 = \boxed{2}$	i) $\log_{11}\left(2\log_4 2^{5!} + \log 10\right)$ $= \log_{11}\left(240 \log_4 2 + 1\right)$ $= \log_{11}(121) = \boxed{2}$

3. Expand and rewrite each expression in terms of log A, log B, and or log C

a) $\log\left(\frac{AB}{C}\right)$ $= \log A + \log B - \log C$	b) $\log\left(A^3 \sqrt{B} \times C^{-1}\right)$ $= 3\log A + \frac{1}{2}\log B - \log C$	c) $\log\left(\frac{10A}{\sqrt{B}}\right) - 0.5\log(100C)$ $= \log 10A - \frac{1}{2}\log B - \frac{1}{2}\log 100C$ $= \log 10 + \log A - \frac{1}{2}\log B - \frac{1}{2}\log 100 - \frac{1}{2}\log C$ $= \cancel{1} + \log A - \frac{1}{2}\log B - \cancel{1} - \frac{1}{2}\log C$ $= \log A - \frac{1}{2}\log B - \frac{1}{2}\log C$

d) $\log\left(0.001\sqrt{ab}\right)$ $= \log 10^{-3} + \log (ab)^{\frac{1}{2}}$ $= -3 + \frac{1}{2}\log a + \frac{1}{2}\log b$	e) $\log\left(\frac{100a}{\sqrt{c^3}b^4}\right)$ $= \log 100 + \log a - \log c^{\frac{3}{2}} - \log b^4$ $= 2 + \log a - \frac{3}{2}\log c - 4\log b$	f) $\log\left(\frac{\sqrt{10b}}{a^{3/2}}\right) - \log c^{-1}$ $= \log 10^{\frac{1}{2}} + \log b^{\frac{1}{2}} - \log a^{\frac{3}{2}} - \log c^{-1}$ $= \frac{1}{2} + \frac{1}{2}\log b - \frac{3}{2}\log a + \log c$
g) $\log\left(\frac{a^{-2}}{1000b^2}\right)$ $= \log a^2 - \log 1000 - \log b^2$ $= 2\log a - 3 - 2\log b$	h) $\left[\log_c\left(\frac{a^2}{b^3}\right)\right]^{-1}$ $= \left(\log_c a^2 - \log_c b^3\right)^{-1}$ $= \frac{1}{\frac{2\log a}{\log c} - \frac{3\log b}{\log c}} = \frac{\log c}{2\log a - 3\log b}$	i) $\frac{\log_a b}{\log_c a} \times \frac{\log_b ac}{\log_b c}$ $\frac{\frac{\cancel{\log b}}{\log a}}{\frac{\log a}{\cancel{\log c}}} \cdot \frac{\frac{\log a + \log c}{\cancel{\log b}}}{\frac{\cancel{\log c}}{\log b}} = \frac{\log a \log b + \log c \log b}{(\log a)^2}$

4. Express each of the following as a single logarithm

a) $2\log a + 5\log b$ $\boxed{\log a^2 b^5}$	b) $3\log x + \frac{1}{2}\log y$ $= \boxed{\log x^3\sqrt{y}}$	c) $2\log a + \log b - 5\log c$ $= \boxed{\log \frac{a^2 b}{c^5}}$
d) $\frac{1}{2}\log x - \frac{2}{3}\log y$ $= \boxed{\log \frac{\sqrt{x}}{\sqrt[3]{y^2}}}$	e) $3\log a - 4\log b - 0.5\log c$ $= \boxed{\log \frac{a^3}{b^4\sqrt{c}}}$	f) $10\log a - \frac{3\log b}{2}$ $\boxed{\log \frac{a^{10}}{b\sqrt{b}}}$
f) $\frac{2\log a}{3\log b} + \frac{5\log b}{2\log b}$ $= \log_b a^{\frac{2}{3}} + \log_b b^{\frac{5}{2}}$ $= \boxed{\log_b \sqrt[3]{a^2}\cdot\sqrt{b^5}}$	g) $\frac{\log a}{0.4\log c} + \frac{\log b}{0.5\log c}$ $= \log_c a^{\frac{1}{0.4}} + \log_c b^{\frac{1}{0.5}}$ $= \boxed{\log_c \sqrt{a^5}\, b^2}$	h) $3\log\sqrt{a} + 2\log\sqrt[3]{b} - 3$ $= \log \sqrt{a^3}\cdot\sqrt[3]{b^2} - \log 1000$ $= \boxed{\log \frac{\sqrt{a^3}\cdot\sqrt[3]{b^2}}{1000}}$

5. If $\log_3 2 = x$, simplify each logarithm in terms of "x"

$a)\ \log_3 8$

$= 3\log_3 2 = \boxed{3x}$

$b)\ \log_3 \sqrt{2}$

$= \log_3 2^{\frac{1}{2}} = \frac{1}{2}\log_3 2 = \boxed{\frac{x}{2}}$

$c)\ \log_3 24$

$= \log_3 3\cdot 2^3 = \log_3 3 + \log_3 2^3 = \boxed{1+3x}$

$d)\ \log_3 18\sqrt{8}$

$= \log_3 2^{\frac{5}{2}}\cdot 3^2 = \frac{5}{2}\log_3 2 + 2 = \boxed{\frac{5}{2}x+2}$

6. Given $\log_3 x = 2$ and $\log_3 y = 5$, evaluate each logarithm

$a)\log_3 xy$

$= \log_3 x + \log_3 y = 2+5 = \boxed{7}$

$b)\log_3\left(27x^3y\right)$

$= \log_3 27 + \log_3 x^3 + \log_3 y = 3 + 3(2) + 5 = \boxed{14}$

$c)\log_3\left(\frac{3x}{y^3}\right)$

$= \log_3 3 + \log_3 x - \log_3 y^3 = 1+2+5(3) = \boxed{-12}$

$d)\log_3\left(810x^{-2}y\right)$

$\log_3 810 - \log_3 x^2 + \log_3 y = 6.096 - 2(2) + 5$

$= \boxed{7.096}$

or $\log_3 10 + 5$

7. Use the fact that $\log_a b = \frac{\log_x b}{\log_x a}$ to simplify the following:

$a)\ (\log_x y)(\log_y x)$

$= \frac{\log_y y}{\cancel{\log_y x}} \cdot \cancel{\log_y x} = \log_y y = \boxed{1}$

$b)\ (\log_5 8)(\log_8 7)(\log_7 5)$

$= \frac{\cancel{\log 8}}{\cancel{\log 5}} \cdot \frac{\cancel{\log 7}}{\cancel{\log 8}} \cdot \frac{\cancel{\log 5}}{\cancel{\log 7}} = \boxed{1}$

$c)\ \left(\frac{1}{\log_d x}\right)\left(\frac{1}{\log_c x}\right)$

$= \frac{\log d}{\log x} \cdot \frac{\log c}{\log x} = \boxed{\frac{\log c \log d}{(\log x)^2}}$

$d)\ (\log_4 a)(\log_a 2a)(\log_{2a} x)$

$= \frac{\cancel{\log a}}{\log 4} \cdot \frac{\cancel{2\log a}}{\cancel{\log a}} \cdot \frac{\log x}{\cancel{2\log a}} = \boxed{\log_4 x} = \frac{1}{2}\log_2 x$

e) $\dfrac{\log_x a}{\log_{xy} a}$

$$= \frac{\frac{\cancel{\log a}}{\log x}}{\frac{\cancel{\log a}}{\log x + \log y}} = \frac{\log x + \log y}{\log x} = \boxed{1 + \log_x y}$$

f) $\dfrac{\log_c m}{\log_{cd} m} - \dfrac{\log_c m}{\log_d m}$

$$= \frac{\frac{\cancel{\log m}}{\log c}}{\frac{\cancel{\log m}}{\log c + \log d}} - \frac{\frac{\cancel{\log m}}{\log c}}{\frac{\cancel{\log m}}{\log d}} = \frac{\cancel{\log c} + \cancel{\log d} - \cancel{\log d}}{\cancel{\log c}}$$

$$= \boxed{1}$$

8. Solve for "y" $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$

$$\frac{\cancel{\log x}}{\log 3} \cdot \frac{\cancel{\log 2x}}{\cancel{\log x}} \cdot \frac{\log y}{\cancel{\log 2x}} = \frac{2\cancel{\log x}}{\cancel{\log x}}$$

$$\log y = 2\log 3 \Rightarrow \log y = \log 3^2 \Rightarrow \boxed{y = 9}$$

9. If $\log x^5 y^3 = 25$ and $\log \dfrac{x}{y} = 3$, then what is the value of $\log x$?

$$\log x^5 + \log y^3 = 25$$

② $5\log x + 3\log y = 25$

① $\log x - \log y = 3$

$$②+3① = 5\log x + \cancel{3\log y} + 3\log x - \cancel{3\log y} = 25 + 9$$

$$8\log x = 34$$

$$\boxed{\log x = \frac{17}{4}}$$

10. Prove the following identity: $\dfrac{1}{\log_a b} + \dfrac{1}{\log_c b} = \dfrac{1}{\log_{ac} b}$

$$\text{LHS} = \frac{\log a + \log c}{\log b} = \frac{\log ac}{\log b} = \boxed{\frac{1}{\log_{ac} b}} = \text{RHS}$$

11. Find the value of $\displaystyle\sum_{n=1}^{999} \log \sqrt[3]{\frac{n^2}{n^2 + 2n + 1}}$

$$\sum_{n=1}^{999} \log \sqrt[3]{\frac{n^2}{(n+1)^2}} = \log \sqrt[3]{\frac{1^2}{\cancel{2^2}} \times \frac{\cancel{2^2}}{\cancel{3^2}} \times \frac{\cancel{3^2}}{\cancel{4^2}} \times \cdots \times \frac{\cancel{999^2}}{1000^2}}$$

$$= \log \sqrt[3]{\frac{1}{1000^2}} = \log \frac{1}{100} = \boxed{-2}$$

12. Simplify the product completely: $\dfrac{\log_2 3}{\log_4 3} \times \dfrac{\log_4 5}{\log_6 5} \times \dfrac{\log_6 7}{\log_8 7} \times \ldots \times \dfrac{\log_{124} 125}{\log_{126} 125} \times \dfrac{\log_{126} 127}{\log_{128} 127}$

$$= \frac{\frac{\cancel{\log 3}}{\log 2}}{\frac{\cancel{\log 3}}{\cancel{\log 4}}} \cdot \frac{\frac{\cancel{\log 5}}{\cancel{\log 4}}}{\frac{\cancel{\log 5}}{\cancel{\log 6}}} \cdot \frac{\frac{\cancel{\log 7}}{\cancel{\log 6}}}{\frac{\cancel{\log 7}}{\cancel{\log 8}}} \cdots \frac{\frac{\cancel{\log 127}}{\cancel{\log 126}}}{\frac{\cancel{\log 127}}{\log 128}}$$

$$= \frac{\log 128}{\log 2} = \log_2 128 = \boxed{7}$$

13. If $\log\left(xy^3\right)=1$ and $\log\left(x^2y\right)=1$. What is $\log\left(xy\right)$? Amc 2003 12A

$\log x^4y^2=2$

$\log xy^3+\log x^4y^2=3 \Rightarrow \log x^5y^5=3 \Rightarrow \log xy=3\cdot\frac{1}{5}=\boxed{\frac{3}{5}}$

14. If $a\ge b>1$, what is the largest possible value of $\log_a\left(a/b\right)+\log_b\left(b/a\right)$? AMC 2003 12B

$=\log_a a-\log_a b+\log_b b-\log_b a$

$=2\underbrace{-\log_a b-\log_b a}_{\text{we want to minimize this value.}}$

the lowest we get is when $a=b$, giving us $2-1-1=\boxed{0}$

15. How many distinct four-tuples (a,b,c,d) of rational numbers are there with:

$a\log_{10}2+b\log_{10}3+c\log_{10}5+d\log_{10}7=2005$ Amc 2005 12B

$\log 2^a3^b5^c7^d=2005$

$2^a3^b5^c7^d=10^{2005}$ $\quad a=c=2005$

$a=c$

$b=d=0$ $\quad$ Only $\boxed{\text{one}}$

16. The numbers $\log\left(a^3b^7\right)$, $\log\left(a^5b^{12}\right)$, $\log\left(a^8b^{15}\right)$ are the first three terms of an arithmetic sequence, and the 12th term of the sequence is $\log\left(b^n\right)$. what is the value of "n"? amc 12

$\log(a^5b^{12})-\log(a^3b^7)=\log(a^8b^{15})-\log(a^5b^{12})$

$\log a^2b^5=\log a^3b^3$

$a^2b^5=a^3b^3$

$\underline{a=b^2}$

$d=\log a^9$

12th term $=a+(n-1)d$

$=\log(b^{13})+11\log b^9$

$=\log b^{13+99}=\log b^{112}$

$\boxed{n=112}$

17. Two distinct numbers "a" and "b" are chosen randomly from the set $\{2,2^2,2^3,\ldots\ldots 2^{26}\}$. What is the probability that $\log_a b$ is an integer? AMC 12 modified

$\log_{2^y}2^x=\dfrac{x}{y}$ $\quad 1\le x,y\le 26$; $x\ne y$

y	# of possible "x"
1	25
2	12
3	7
4	5
5	4
6	3
7	2
8	2
9	1
10	1
11	1
12	1
13	1
⋮	⋮
	0

$=65$ total cases

$P()=\dfrac{\text{Desired}}{\text{Total}}=\dfrac{65}{26\times 25}=\boxed{0.1}$